Basics of a Motor

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Content

Basic Terminology

1. Stator & Rotor

   Generally, stator is the stationary part and rotor is the rotational part. In case of BLDC, often stator will have windings and rotor will have the permanent magnets.

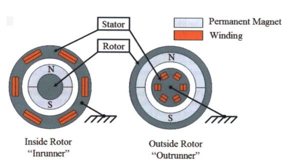

Figure 1: Stator & Rotor, Source:[1]

2. Air Gap

   Air Gap is the distance between stator and rotor.

3. Tooth & Slot

   Winding are wounded around the laminated steel structure called tooth which channels more magnetic flux through them.

   Slot is the section between two tooth. Three phase motor have slots (and teeth) that is evenly divisible by three.

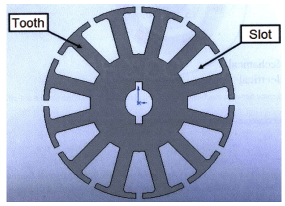

Figure 2: Tooth & Slot, Source:[1]

4. Phase

   A phase is an individual group of windings with a single terminal accessible from outside the motor. Three phase windings have been used traditionally because of simplicity and over time it has become the industry standard. However, using windings with an increased number of phases will reduce torque ripple further and increase motor efficiency [46]. Currently, a custom 8-phase brushless motor is being developed for the MIT Cheetah that will have significantly higher torque density, **reduced torque ripple, and higher efficiency in the low-speed, high-torque operating regime** as compared to COTS motors [51].

5. Turns

   Each individual loop of wire making up phase winding is a Turn.

6. Pole (N_m)

   A pole is a single permanent magnet pole, north or south. The minimum number of poles is two, but motors can have any even number of poles.The number of poles is not directly related to the number of slots, although there are common combinations of slot and pole counts that work well [6].

7. Field windings and Armature winding

   The windings which are responsible for the Magentic Field (**B**) is the field winding. The energy we apply here merely act as catalyst alone.

   Armature winding consist of current carrying conductors , which experience the BIL force, thus are electrical power applied here is converted into mechanical output power.

8. Electrical Angle (θ_e) and Mechanical Angle (θ_m)

   Mechanical degrees (θ_m) in a motor refers to the rotation of the shaft. 1 revolution of the shaft equals 360 mechanical degrees. When an electrical machine is operating as a motor, the idea is to create a traveling, rotating magnetic field, via the stator so that this moving flux attracts the rotor. Electrical degree (θ_e) is the angle through which magnetic field has rotated, i.e., 360 electrical degree equals transistion form “North” to “South” to “North”

A 2 pole motor has 1 “North” pole and 1 “South” pole on the rotor. So in order for it to turn 360 electrical degrees (“North” to “South” to “North”), it needs to rotate 360 mechanical degrees. A 4 pole motor has 2 “North” poles and 2 “South” poles. That means that 360 electrical degrees will occur when the shaft has rotated only 180 (360/2) mechanical degrees. Thus for N_m Poles we have, $\theta_e = \frac{N_m}{2}\theta_m$

Motor Parameters

Parameterisation means, to develop the model of a “system” interms of paramters(variable) and to see how various parameters affect the “system”. The developed model should be verified by comparing the theoritical prediction form the model against experimental data and based upon the error, we should improve our model till we are satisfied with model’s accuracy. Thus before delving into the various paramters of a motor, we need to have model the motor. Most motor are modelled as a Resistor, Inductor and volatge source(E) in series:

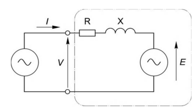

Figure 3: Basic Motor Model, Source:[1]

For a 3-phase BLDC/PMSM, the appropriate model is:

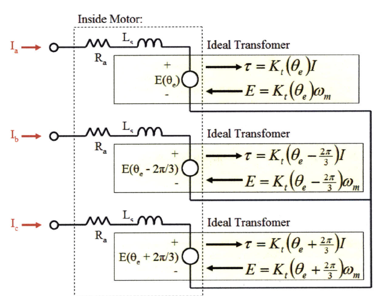

Figure 4: Model of a 3-phase BLDC/PMSM, Source:[1]

Electrical

1. Resistance

   Responsible for heating loss, can be easily modeled as simple lumped resistor.

2. Inductance

Motor windings have inductance. Physically, this means that current flowing in the windings will induce magnetic flux through them, even in the absence of permanent magnet flux and will resist rapid changes in current by generating voltage across it. However, this is not the back EMF.The value of inductance is less straightforward to calculate because the phases are not magnetically independent. That is, current in one phase can induce flux in another. Under sinusoidal drive currents, it is possible to use a lumped inductance, called the **synchronous inductance**, to accommodate for this. The value of the synchronous inductance is: [1] $L_s = \frac{3}{2}L_a$ where L_a is the lumped inductance measured independently on one phase, if it could be isolated. The winding inductance stores energy in the form of a magnetic field any time there is current in the winding. When a winding is switched off, this energy must go somewhere. For this reason, controller drivers contain **flyback diodes** that allow this current to circulate even when all the switches are open. Under highfrequency pulse-width modulated (PWM) control, the winding inductance also filters out current ripple. However, as a low-pass filter on current it also creates phase lag which is the motivation for the use of field-oriented control. The winding inductance is a function of motor geometry and the number of turns in the winding.

Saliency

Saliency means the inductance varies with rotor position due to non-uniform air gap which inturn creates non-uniform flux distribution. If the magents are removed, the rotor will align with ampere-conductor distribution of stator and the torque produced for alignment is called **alignment/reluctant** torque. In Non-Salient, the rotor is rotationally symmetric and has no tendency to align with stator if, magnets in rotor are removed. i.e., No **Reluctance Torque** thus, winding inductance does not vary with rotor position. Non-salient poles exhibit both attractive & repulsive gap forces. [2, p. 68]

Mechanical

1. Winding Configuration

   They have three phase windings and can be connected to each other in wye or delta configuration. Wye has higher torque (theoretically torque constant is greater by a factor $\sqrt{3}$ [3, p. 25]) because in the wye configuration, at any time one of phase is open and other two-phase are in series thus equal current flows through them whereas in delta it gets divided into two phases. Thus more current in each phase causes more torque, one will push and another phase will pull in the wye configuration.

2. Gap Radius:

Gap Radius R_{gap} is radial distance from the axle to the gap between the stator and rotor.

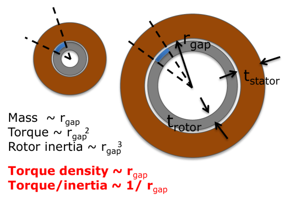

Figure 5: Gap Radius, Source:[4]

When we mean pancake shaped motor, we mean motors with large R_{gap} beacause it will increase torque per unit mass but it doesn't mean we can increase the R_{gap} indefinitely beacause that will decrease the torque per unit inertia. Thus a compromise has to be made and mostly $R_{gap} > L_{axial}$. In general outrunners will be preferrable to innrunners due to large gap radius.

Derivation:

$$Mass = V\rho = [V_{rotor} + V_{stator}]\rho$$

$$M = \pi[r_{gap}^2 - (r_{gap} - t_{rotor})^2 + (r_{gap} + t_{stator})^2 - r_{gap}^2]l\rho$$

Approximating to First Order thin walls (Neglecting higher order terms)[5]

$$M \approx 2\pi r_{gap} l(t_s\rho_s + t_r\rho_r)$$

$$M \propto r_{gap} \tag{1}$$

$$\tau = Fr_{gap} = (\sigma A)r_{gap} = (\sigma 2\pi r_{gap} l)r_{gap} = 2\pi r_{gap}^2 l\sigma$$

$$\tau \propto r_{gap}^2 \tag{2}$$

$$J_{hollowcylinder} = \frac{1}{2}M(R_{inner}^2 + R_{outer}^2)$$

$$J_{rotor} \approx Mr_{gap}^2$$

$$J_{rotor} \approx 2\pi l t_r \rho_r r_{gap}^3$$

$$J_{rotor} \propto r_{gap}^3 \tag{3}$$

Frome eq. 1, eq. 2, eq. 3

$$\frac{\tau}{M} \propto r_{gap}$$
$$\frac{\tau}{J} \propto \frac{1}{r_{gap}}$$

3. Core vs Coreless
4. Axial Flux vs Radial Flux

   [**6**] Force Control and Position Control are two ends of extreme. Force control sensible in case of contact. Position control sensisble in case of contact-less

   Thus for a real world we need a hybrid approach

Characterisation

Constant

1. Torque/Back EMF constant (K_t)

   It measure of how much torque (resp. back-emf) can be produced given my current (resp. speed) and vice-versa.

$$T = K_t i$$

$$E = K_e \omega_m$$

   where K_t = Torque constant; K_e = Back emf constant; T =Torque; E =Back EMF; i =current; ω_m =angular speed;

   Theoretically $K_t = K_e$ but in reality, K_e can only be measure in open-circuit and K_t can only measured when current is flowing, thus causisng a variance in magentic and electrical condition which casuses **different value** to each of them. They are determined by magnetic field and geometrical parameters of air gap. For an ideal motor it will remain constant, but in reality, it varies with rotor position, i.e, $K_t = f(\theta_e)$ because magetic field (B) consist of discrete poles and commutation occurs discretely which causes ripple in magnetic field. To reduce the torque ripple we can increase switching rate of current, make the current to be smooth and increase the number of poles.

Derivation:

Electrical input power = rate of production of *heat* in conductor + power absorbed by the inductor(*magnetic* energy) + *mechanical* output power

$$V_{supply} i = i^2 R + iL\frac{di}{dt} + (Bil)\upsilon$$

Under steady-state condition (accelration is zero), $\frac{di}{dt} = 0$:

$$V_{supply} i = i^2 R + (Bil)\upsilon$$

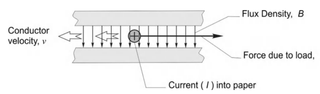

Figure 6: primitve motor in a steady-state, Source:[7]

When $\upsilon = 0$, there is no mechnical output power, and all the electrical energy will converted as heat loss:

$$V_{supply,\upsilon=0} i = i^2 R$$

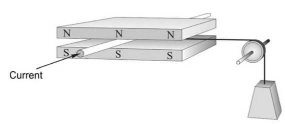

Figure 7: primitve motor in a stall condition, Source:[7]

However current i will remain unchanged for both $\upsilon \neq 0$ & $\upsilon = 0$, because it is determined by the load alone, thus to support a given load the current i is:

$$F_{conductor} = m_{load} g = Bil_{axial} \implies i = \frac{m_{load} g}{Bl} \implies i \propto m_{load}$$

since (g, B, l) are constant for a given motor.

To move the load, i.e., to produce mechnical output power, we need higher volatge, i.e., $V_{supply} > V_{supply,\upsilon=0}$

$$(V_{supply} - V_{supply,\upsilon=0}) i = (Bil_{axial})\upsilon$$

$$V_{supply} - V_{supply,\upsilon=0} = Bl_{axial}\upsilon = E$$

where E is the extra voltage needed to move the load, which proportional velocity of the conductor relative to the flux, for give field(B) & load(I) and is known as **Back EMF/Motional EMF**, thus:

$$Ei = T_{electromagnetic}\,\omega_m$$

where $T_{electromagnetic} = T_{shaft} + frictional_{loss}$

$$\implies T = K_t i$$

$$\implies E = K_e \omega_m$$

To avoid the torque ripple, we can:

2. Motor Constant K_m

   It is the ability to produce Toqrue for a given input power. It is winding invariant [8] as long as same conductimg wires are used. (i.e., $K_m \propto r_{wire}^0$) and the square of motor constant is directly proportional to cube of r_{gap}

$$K_m = \frac{K_T}{\sqrt{R}} = \frac{T}{\sqrt{P_{input}}}$$

   Derivation:

$$P_{input} = Vi = i^2 R = \left(\frac{T}{K_t}\right)^2 R = T^2 \frac{R}{(K_t)^2}$$

$$\implies \frac{(K_t)^2}{R} = \frac{T^2}{P_{input}}$$

$$\implies K_m = \frac{K_T}{\sqrt{R}} = \frac{T}{\sqrt{P_{input}}}$$

   For given winding volume, ($V = \pi r_{wire}^2 L = c \implies L \propto \frac{1}{r_{wire}^2}$), the resistance of the winding, $R \propto \frac{L}{A} \propto \frac{L}{r_{wire}^2} \propto \frac{1}{r_{wire}^4}$. The Torque constant K_t depends linearly on the number of turns around the core, thus:

$$K_t \propto L \propto \frac{1}{r_{wire}^2}$$

$$\tau = K_t i \implies \tau \propto \frac{i}{r_{wire}^2} \tag{4}$$

$$P = i^2 R \propto \frac{i^2}{r_{wire}^4} \tag{5}$$

From eq. 4 and eq. 5

$$\implies K_m = \frac{\tau}{\sqrt{P}} \propto r_{wire}^0$$

$$K_m^2 = \frac{\tau^2}{i^2 R} = \frac{n(B^2 i^2 l_{axial}^2 r_{gap}^2)}{i^2 \rho \frac{l_{axial}}{A}} = \frac{n l_{axial} B^2 r_{gap}^2 A}{\rho}$$

thus for given a particular wire gauge, the number of wires (n) in the cross section scales linearly with the radius.[5]

$$\implies K_m^2 \propto r_{gap}^3$$

3. Electric Time Constant

   The electrical time constant is the amount of time it takes the current in the winding to reach 63.2% percent of its rated value. The time constant found by dividing inductance by resistance.[9]

$$\tau_e = \frac{L}{R}$$

4. Mechanical Time Constant

   It is the time required for a given motor to reach 63.2% of its maximum rated speed in a no-load condition. The mechanical time constant is basically a measure of a motor's responsiveness. Direct Drive has no damping, thus is τ_{mech} large comapred to geared drive.

$$\tau_{mech} = \frac{inertia\ load}{mechanical\ damping\ coefficient} = \frac{\sum R_a J_t}{K_e K_t}$$

5. Thermal Specific Torque Density K_{ts}

   This measure describes a motor's ability to produce torque at stall while the windings dissipate energy through Joule heating corresponding to $100 \deg C$ rise [10].

$$K_{ts} := \frac{K_t}{m}\sqrt{\frac{1}{R_{th}R}}$$

   where K_t is the torque constant $(\frac{Nm}{A})$, m is the motor mass (kg), R_{th} is the motor thermal resistance $(\frac{\deg C}{W})$ and R is the winding electrical resistance in ohm.

Loading

1. Specific Electrical Loading $(\overline{A})$

   Avergae axial current per meter of circumference on the rotor. The current is concentarted in the conductors between the slot.

2. Specific Magnetic Loading $(\overline{B})$

   Average radial flux density over the entire cylindrical surface of the rotor. density over the entire cylindrical surface of the rotor. Because of the slotting, the average flux density is always less than the flux density in the teeth.

   $(\overline{A})$ **vs** $(\overline{B})$

   In case we widen our slots to accomodate more copper/conductor $(\overline{A})$ will increase but $(\overline{B})$ may reduce because narrower teeth means less area for flux, and therefore has danger of stauration of iron. Thus a compromise has to be made for a given a volume.

3. Thermal Load (C)

   As intuitive and as basic the better the cooling the better will be the perfomance of our motor.

Losses

Losses can be majorly grouped into two general categories:

1. Torque/Copper/I^2 Losses

   They are due to resistance heating of the windings, and are proportional to the square of the output torque.

2. Speed/Iron/E^2 Losses

   Iron Core Losses = Eddy Curremt Losses + Hystersis Losses

   Hysteresis in the magnetic domains in the core appears as a constant friction torque, while the eddy current torque is proportional to speed.

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